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A RECONSIDERATION OF THE TRANSITION CRITERION FROM MACH TO REGULAR REFLECTION OVER CYLINDRICAL CONCAVE SURFACES

K. Takayama* and G. Ben-Dor**

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Following the idea forwarded by Ben-Dor and Takayama(1985) a new propagation path was assumed for the corner generated signals. In addition to the new chosen propagation path, one of the simplified assumptions used by Ben-Dor and Takayama, namely that $u+a$ remains constant behind the incident shock wave, was further simplified, i.e., both u and a were assumed to be constant behind the incident shock wave. This new path and the further simplified assumption led to a new transition criterion from Mach to regular reflection over a cylindrical wedge which unlike the two criteria developed in Ref. by Ben-Dor and Takayama(1985) has the ability to predict the transition wedge angle quite accurately in the entire range of the incident shock wave Mach numbers which were investigated in Ref. by Ben-Dor and Takayama(1985) and in this study.

Key Words: Reflection, Shock Wave, Concave Surface, Blast Wave, Mach Reflection

1. INTRODUCTION

When an explosion generated blast wave encounters a structure it reflects over it initially either as a regular reflection or as a Mach reflection depending upon the geometrical shape of the structure. If the structure has a concave shape, such as the one shown in Fig. 1, then the initial reflection will be a Mach reflection (MR) which will eventually transition into a regular reflection (RR)(Ben-Dor et al., 1980). The exact location of the MR→RR transition along the structure surface is very important if one wishes to calculate the blast loading on the structure, since the flow fields resulted by a MR or a RR are different.

In a previous study(Ben-Dor and Takayama, 1985) two possible analytical criteria for predicting the transition from Mach to regular reflection over cylindrical concave wedges were developed. Both criteria were derived using the hypothesis of Hornung, Oertel, and Sandeman(1979), that a Mach reflection can exist only if the corner generated signals can catch up with the incident shock wave, and hence communicate a physical length scale to the reflection point. Without this length scale a Mach reflection which is typified by a shock wave with a finite length, the Mach stem, is impossible.

The difference between the two criteria which were developed in Ref. (Ben-Dor and Takayama, 1985) was the assumed propagation path of the corner generated signals. When the propagation path was assumed to be along either side of the slipstream [model A in Ref. (Ben-Dor and Takayama, 1985)] the following transition criterion was obtained:

$$\frac{\sin\theta_W^*}{\theta_W^*}=\frac{M_i}{U_{10}+A_{10}} \tag{1}$$

*Institute of High Speed Mechanics Tohoku University, Sendai, Japan
**Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, 84 105(P.O.B 653), Israel

where θ_W^* is the wedge angle at which transition from Mach to regular reflection takes place, M_i is the incident shock wave Mach number, $U_{10}=u_1/a_0$ and $A_{10}=a_1/a_0$ where u is the shock induced flow velocity, a is the local speed of sound, and subscripts "0" and "1" define the flow states ahead and behind the incident shock wave-i. When the corner generated signals were assumed to travel along the shortest possible path, i.e., a straight line connecting the leading edge of the reflecting wedge and the transition point (model B in Ref. Ben-Dor and Takayama, 1985) the obtained transition criterion was:

$$\cos\frac{1}{2}\theta_W^*=\frac{M_i}{U_{10}+A_{10}} \tag{2}$$

Note that both criteria were developed for a constant velocity planar shock wave reflecting over a cylindrical concave surface. This case is simpler than the one encountered when a blast wave reflects over a cylindrical concave surface, for a blast wave attenuates as it propagates outwards. Thus, the above mentioned study as well as the one given subsequently should be regarded as a first order solution only.

These two transition criteria (Eqs. 1 and 2) are plotted in Fig. 2 as curves A and B, respectively. The experimentally

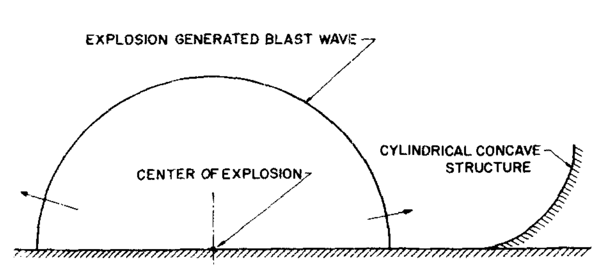

Fig. 1 A schematical illustration of an explosion generated blast wave propagating towards a cylindrical concave structure

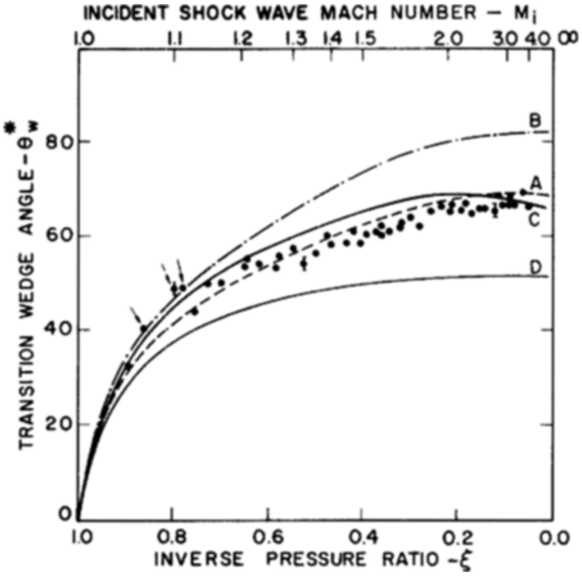

Fig. 2 The transition wedge angle, θ_w^*, from Mach to regular reflection over a cylindrical concave wedge as a function of the inverse pressure ratio-ξ across the incident shock wave, (lower scale) or its Mach number-M_i(upper scale)
A-Model A of Ref. (Ben-Dor et al., 1980)
B-Model B of Ref. (Ben-Dor et al., 1980)
C-The present Model
D-The detachment criterion for transition over a straight wedge

measured transition wedge angles as taken from Ref. (Ben-Dor et al., 1980) are also included in the (θ_w^*, ξ)-plane shown in Fig. 2. The recorded transition wedge angles are accurate to about $\pm 1.5^\circ$: Typical error bars are shown in some of the experiments. Curve D is the well known "detachment" transition line which is applicable to pseudo-steady flows only, i.e., reflection of constant velocity planar shock waves over straight wedges.

As can be seen, the agreement between the experimental results and the transition line predicted by Eq. (1) is quite good in the range $0.76 > \xi > 0.05$ which is equivalent to the range $1.125 < M_i < 4$(the upper limit arises from lack of experimental results). At the lower Mach number range, $1 < M_i < 1.125$($1 > \xi > 0.76$), the agreement between the theory(curve A in Fig. 2) and the experiments is very poor(see the three experiments marked with an arrow which lie 4.5° to 7.5° above curve A.) The second criterion, however, (curve B in Fig. 2) resembles excellent agreement with these experiments only, i.e., it is good only in the range $1 < M_i < 1.125$($1 > \xi > 0.76$). Beyond this narrow range, the analytical predictions based on Eq. (2) are 10° to 15° greater than those obtained experimentally.

Thus it can be seen that the two transition criteria, which differ in the assumed propagation path of the corner generated signals, fail to accurately predict the transition wedge angle in the entire investigated range of the incident shock wave Mach number.

For this reason it was decided to re-consider the solution outlined in Ref. (Ben-Dor and Takayama, 1985), and try to obtain a better transition criterion which will be useful in the entire Mach number range.

2. PRESENT STUDY

In order to improve the analytical prediction of the transition from Mach to regular reflection over concave cylinders the propagation of the corner generated signals was reconsidered.

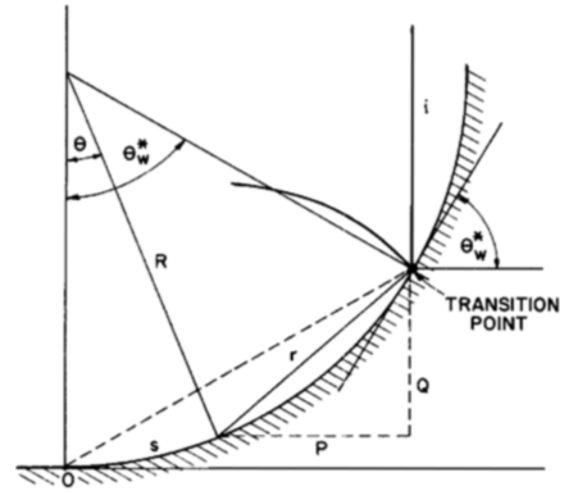

Fig. 3 A schematical illustration of a Mach reflection over a concave cylindrical wedge exactly at transition. O-leading edge of the wedge, R-radius of curvature of the wedge, θ_w^* transition wedge angle, s-distance along the reflecting wedge travelled by the gas particle from $t=0$, r-radius of the sonic disturbance produced by the travelling gas particle and i-incident shock wave

Figure 3 illustrates a Mach reflection exactly at transition. The triple point touches the reflecting wedge surface and hence the Mach stem has completely disappeared. Let us follow a gas particle which propagates along the concave wedge surface. If it is the time measured from the moment the incident shock wave, i, encountered the leading edge of the reflecting wedge (point O), then the distance travelled by the gas particle along the curved surface is

$$s = \int_0^t u \, dt \tag{3}$$

where u is the particle velocity. The disturbance generated by this particle propagates with the local speed of sound, a. The area which has been reached by this disturbance is bounded by a circle having radius r which can be obtained from.

$$r = \int_0^t a \, dt \tag{4}$$

Note that when the incident shock wave is weak enough the reflected shock wave near the triple point coincides with this sonic circle. Thus, in these cases the reflected shock wave is a part of the envelope of this disturbance.

The foregoing discussion suggests that the distance to which the corner generated signals have propagated can be obtained by a vector summation of the particle path, s, and the disturbance path, r, as indicated by the dashed line in Fig. 3.

In Ref. (Ben-Dor and Takayama, 1985) it was assumed that $u + a = u_1 + a_1$ where subscript "1" denotes the flow state behind the incident shock wave. Unlike this assumption let us assume for the present case that: $u = u_1$ and $a = a_1$. Note that the assumption in Ref. (Ben-Dor and Takayama, 1985) was based on the experimental fact that the reflected shock wave

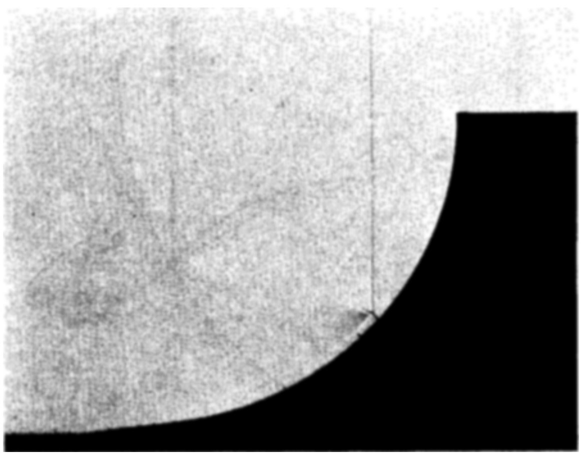

Fig. 4 A typical shadowgraph of a Mach reflection over a concave cylinder. Note how the reflected shock wave becomes weaker and weaker as it approaches the surface near the leading edge of the reflecting surface

appears to be very weak near the leading edge of the reflecting wedge. A typical Mach reflection is shown in Fig. 4. The fact that the reflected shock wave almost vanishes near the leading edge of the reflected wedge is clearly visible. For this reason it can be assumed to behave as a Mach wave (degenerated shock wave) across which the gas properties remain constant. Using this assumption Eqs. (3) and (4) can be simply integrated to obtain.

$$s = R\theta = u_1 t \quad (5)$$
$$r = a_1 t \quad (6)$$

where R is the radius of curvature of the cylindrical wedge and θ is an angle indicating the angular position along the reflecting wedge reached by the gas particles during the time, t.

Inspecting Fig. 3, one can easily get

$$(R\sin\theta_w^* - R\sin\theta)^2 + (R\cos\theta - R\cos\theta_w^*)^2 = r^2$$

which with the aid of Eq. (6) can be rewritten as:

$$(\sin\theta_w^* - \sin\theta)^2 + (\cos\theta - \cos\theta_w^*)^2 = \left(\frac{a_1 t}{R}\right)^2 \quad (7)$$

from Eq. (5) we have

$$\theta = \frac{u_1 t}{R} \quad (8)$$

and if the incident shock wave velocity is u_1 then one can write

$$\sin\theta_w^* = \frac{u_1 t}{R} \quad (9)$$

Inserting Eq. (9) into Eqs. (7) and (8) while elimating the term t/R results in

$$\theta = \frac{u_1}{u_i}\sin\theta_w^* \quad (10a)$$
$$\frac{2-2\cos(\theta-\theta_w^*)}{\sin^2\theta_w^*} = \left(\frac{a_1}{u_i}\right)^2 \quad (10b)$$

Eq. (10) can be rewritten as:

$$\theta = \frac{U_{10}}{M_i}\sin\theta_w^* \quad (11a)$$
$$\frac{2-2\cos(\theta-\theta_w^*)}{\sin^2\theta_w^*} = \left(\frac{A_{10}}{M_i}\right)^2 \quad (11b)$$

where $U_{10} = u_1/a_0$, $A_{10} = a_1/a_0$ and $M_i = u_i/a_0$ is the incident shock wave Mach number.

Using simple trigonometric relations Eq. (11b) can be further simplified to read:

$$\frac{2\sin\dfrac{\theta_w^*-\theta}{2}}{\sin\theta_w^*} = \frac{A_{10}}{M_i} \quad (11c)$$

As shown in Ref. (Ben-Dor and Takayama, 1985), for a perfect gas with a given specific heat capacities ratio, γ, both U_{10} and A_{10} depend solely on the incident shock wave Mach number M_i through the following relations;

$$U_{10} = \frac{2(M_i^2-1)}{(\gamma+1)M_i}$$
$$A_{10} = \frac{\gamma-1}{\gamma+1}\frac{1}{M_i}\left[\left(\frac{2\gamma}{\gamma-1}M_i^2-1\right)\left(M_i^2+\frac{2}{\gamma-1}\right)\right]^{1/2}$$

Thus for a given incident shock wave Mach number, M_i, Eqs. (11a & c) can be solved using an iterative method, to obtain θ and θ_w^*.

3. RESULTS AND DISCUSSION

The transition wedge angle, θ_w^*, as obtained from the iterative solution of Eqs. (11a & c) is shown in Fig. 2 as curve C. Unlike the previous transition lines A and B which were good only at $M_i > 1.125$ and $M_i < 1.125$, respectively and were poor beyond these ranges, the new transition line showns in general a fairly good agreement with the experimental results in the entire range of M_i. In the range $1 < M_i < 1.25 (1 > \xi > 0.6)$ it lies slightly below curve B and resembles a very good agreement with the experiments marked with an arrow that were very poorly predicted by curve A. In the range $1.25 < M_i < 2.00 (0.6 > \xi > 0.22)$ the predictions of the presently developed transition criterion are up to about 5° too high while for $M_i > 2.00 (\xi < 0.22)$ the agreement becomes again good. Note that in the range $1.125 < M_i < 2.00$ the predictions of curve A are better than those of curve C. But as mentioned earlier at the lower Mach regime the predictions of curve A are very poor. Thus, curve C is superior for it is capable of predicting the transition wedge angle over the entire investigated range of incident shock wave Mach numbers-M_i.

4. CONCLUSION

Using a slightly different approach than the one used in Ref. (Ben-Dor and Takayama, 1985) a new analytical criterion for the transition from Mach to regular reflection over a cylindrical concave wedge was developed. Unlike the assumption used in Ref. Ben-Dor and Takayama, 1985 that $u + a = \text{Const.}$ in the entire flow field behind the incident shock wave, in the present approach it was assumed that each u and a are constant in the flow field behind the incident shock wave. This simplifying assumption led to a new transition criterion which is capable of predicting the transition wedge angle in the range of $1 < M_i < 4$.

The fact that in the range $1.125 < M_i < 2.00$ one of the previous criteria(curve A) is better than the present one(curve C) should not discard the present approach since in reality the flow velocity, u, along the wedge is smaller, due to the compression effect, than the shock induced velocity, u_1, and therefore, if a more realistic distribution of the flow velocity, u, was used, then both curves A and C would shift downwards. In such a realistic case curve C would resemble a better agreement with the experimental results while the predictions of curve A would become poorer. Unfortunately, however, a physical model by which the flow field could be better approximated is unavailable. It should also be noted that the present solution as well as those given in Ref. (Ben-Dor and Takayama, 1985) are for an invicid gas. However, it is well known from pseudo-steady reflection experimental studies that viscous effects cause transition angles which are up to 5° lower than those predicted analytically. Thus one assumes that the inclusion of viscous effect into the present model would have a similar effect then again the predictions of curve C would improve while those of curve A would become worse. Unfortunately, the inclusion of viscous effects in a simple way is impossible.

Thus, it is concluded that the present approach is probably valid but better information of the distribution of both u and a along the wedge surface is needed in order to have a better agreement with the experimental results.

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